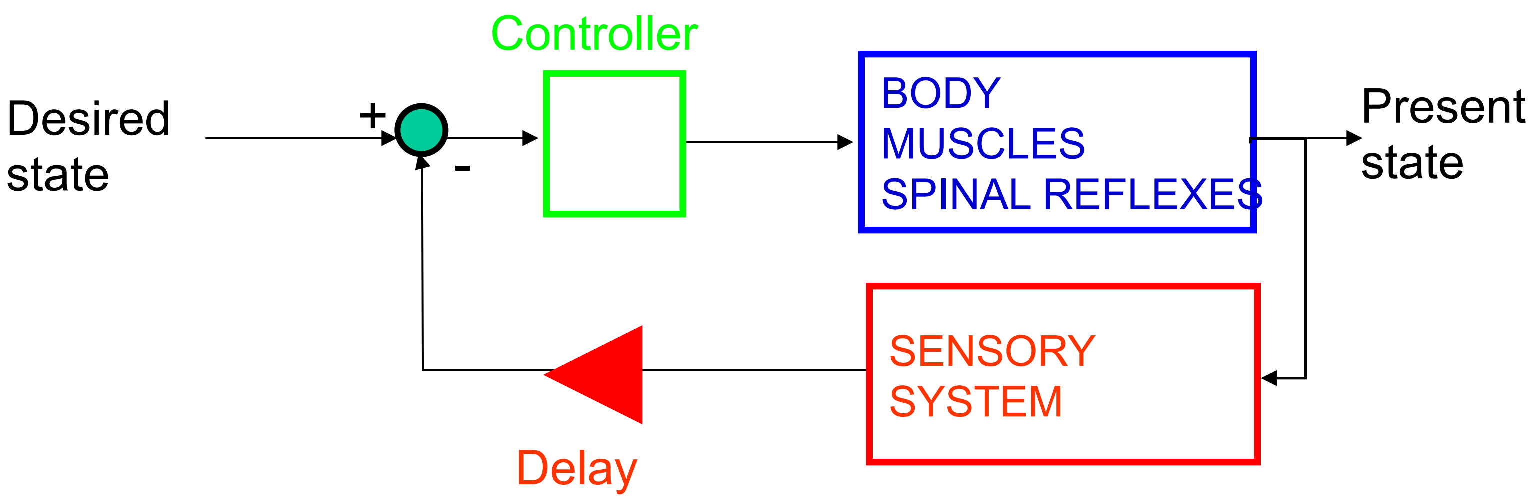
D4. Computational motor control

Using sensory information in movements

Perception vs action

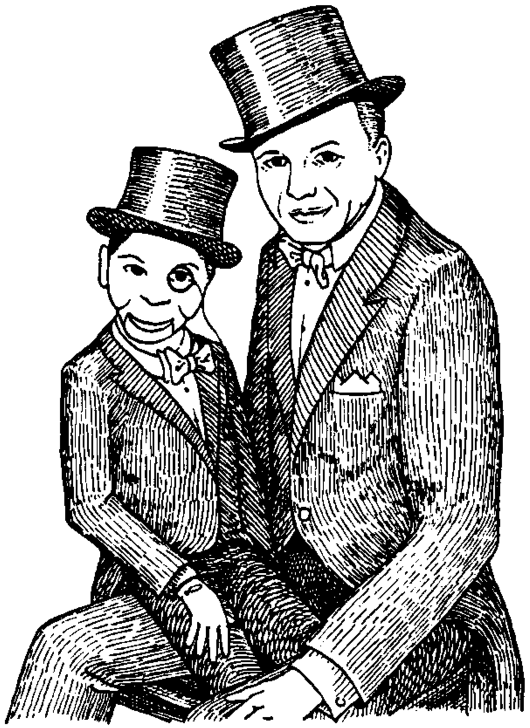


•Sensory information is made available with a delay that has the same order of magnitude of movement time constants

•To have high performance (high speed of correction), gain must be high…

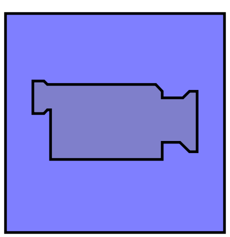
•Beyond a certain gain, delays make the closed-loop system unstable

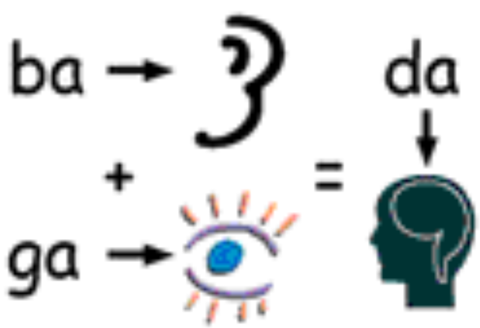
Multi-sensory integration



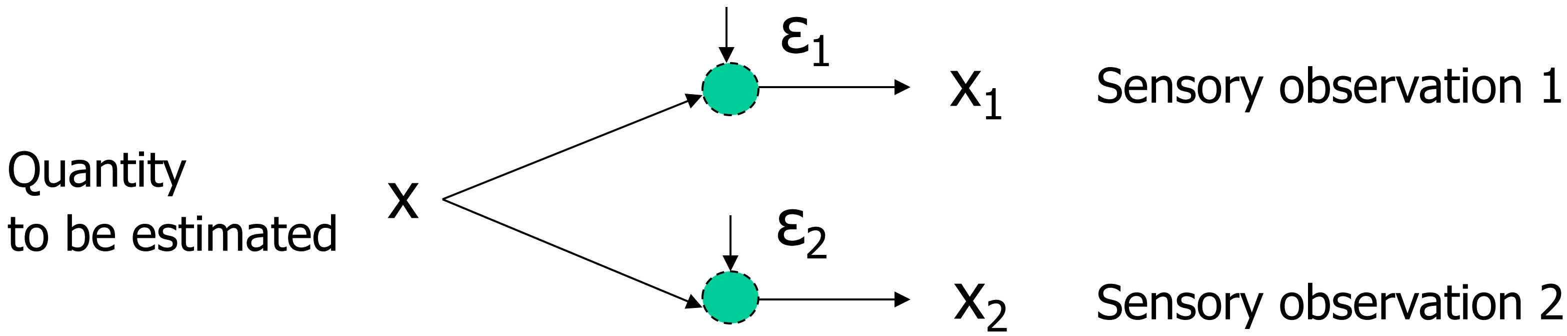
- One of main functions of nervous system
    - Ventriloquists
    - McGurk effect

- What for?
    - Redundancy
      To reduce uncertainty
    - Complementarity
        - To get information that is not present in one single sensory modality (e.g., stereo vision)
    - Processing speed
      Speed vs accuracy trade-off





The multi-sensory integration problem



$$\hat{x} = \arg\max_x P(x|x_1, x_2) \propto P(x_1, x_2 \mid x) \cdot P(x)$$

$$\hat{x} = \frac{\sigma_1^{-2}}{\sigma_1^{-2} + \sigma_2^{-2}} x_1 + \frac{\sigma_2^{-2}}{\sigma_1^{-2} + \sigma_2^{-2}} x_2$$

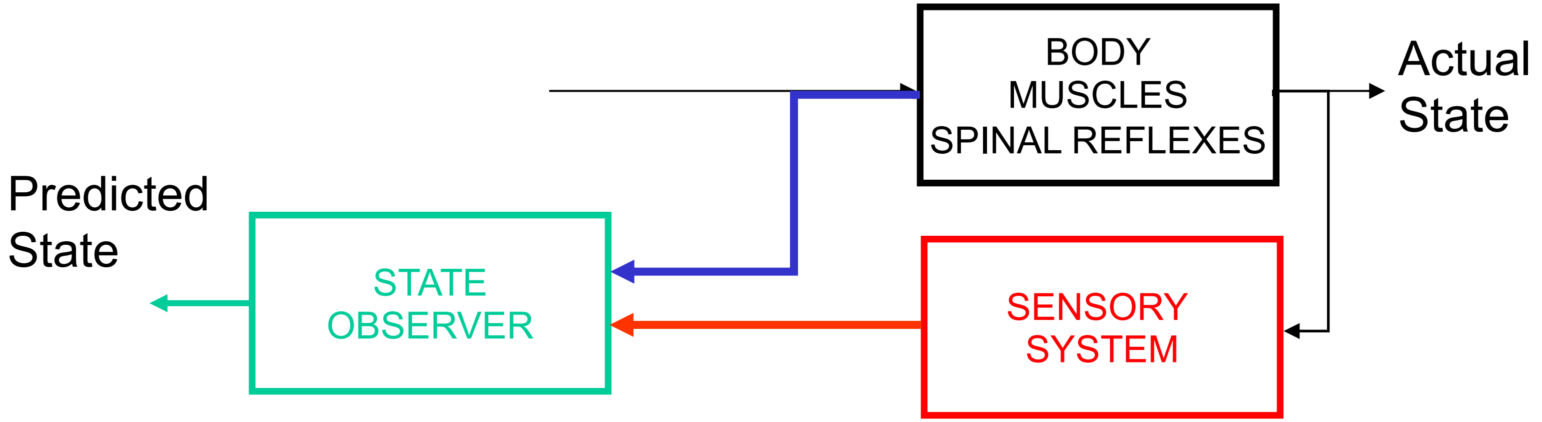
- Solution: Weight more the observation that is most reliable
- What if the observations are biased?

Sensorimotor integration

Suppose to move arm in the dark.
At least 2 sources of information are available to estimate its state (position, speed):

- sensory afferences (**reafferences**)
- motor commands (**efferent copies**)

- Still an integration problem, with two additional aspects:
  - The arm moves, therefore the estimate has to be kept up-to-date
  - Motor commands need to be 'integrated' (dead-reckoning)



Sensorimotor integration

Assume that motion equations can be approximated by a linear system dynamics:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + w_k \\ y_k = Cx_k + v_k \end{cases}$$

Where w_k and v_k are, respectively, **process** and **measurement** noise which are assuemd to be independent, Gaussian, uncorrelated and zero-mean:

$$\begin{cases} p(w_k) \sim N(0, Q) \\ p(v_k) \sim N(0, R) \end{cases}$$

NB: Musculoskeletal system is a physical system, described by continuous-time equations… We use a discrete-time formulation because it is useful in simulations

Similar conclusions may be drawn from continuous-time simulations

Terminology: prior and posterior errors

Let us define:

$\hat{x}_{k+1}^{-}$ **Prior** estimate of x_{k+1}, given the knowledge of the process (input, utput) until time step k

$\hat{x}_{k+1}$ **Posterior** estimate of x_{k+1}, given the knowledge of the process until time step k and measurement y_{k+1} at time step (k+1)

$e_{k+1}^{-} = x_{k+1} - \hat{x}_{k+1}^{-}$ Prior estimation error

$e_{k+1} = x_{k+1} - \hat{x}_{k+1}$ Posterior estimation error

$P_{k+1}^{-} = E\left\{ e_{k+1}^{-} \cdot e_{k+1}^{-\,T} \right\}$ (Prior) covariance of e_{k+1}^{-}

$P_{k+1} = E\left\{ e_{k+1} \cdot e_{k+1}^{T} \right\}$ (Posterior) covariance of e_{k+1}

Kalman filter

- Poterior state estimate can be expressed as a linear combination of prior state estimate and the difference between actual and predicted measure (sensory prediction error):

$$\hat{x}_{k+1} = \underbrace{\hat{x}^-_{k+1}}_{\text{predizione}} + \underbrace{K_{k+1} \cdot \left[y_{k+1} - C \cdot \hat{x}^-_{k+1} \right]}_{\text{innovazione o residuo}}$$

- where:

$$\hat{x}^-_{k+1} = A\hat{x}^-_k + Bu_k$$

- This estimate is unbiased (posterior estimation error has zero mean), must find K in order to get minimum variance…

Kalman filter: derivation

- Quantity K (aka Kalman gain) can be obtained (similar to multisensory integration) by maximising the probability of the posterior state estimate…
- We finally get:

$$K_{k+1} = P_{k+1}^{-} C^{T} \cdot \left(C P_{k+1}^{-} C^{T} + R \right)^{-1}$$

- **Consequences:**
  1. If covariance of measurement error, R, tends to zero (perfect sensing), innovation has maximum weight:

$$\lim_{R \to 0} K_{k+1} = C^{-1}$$

  2. If covariance of prior state estimate is zero (perfect prediction), innovation has zero weight:

$$\lim_{P_{k+1}^{-} \to 0} K_{k+1} = 0$$

- Therefore, as in multisensory integration, factors are weighted in terms of their reliability…

Sensorimotor integration

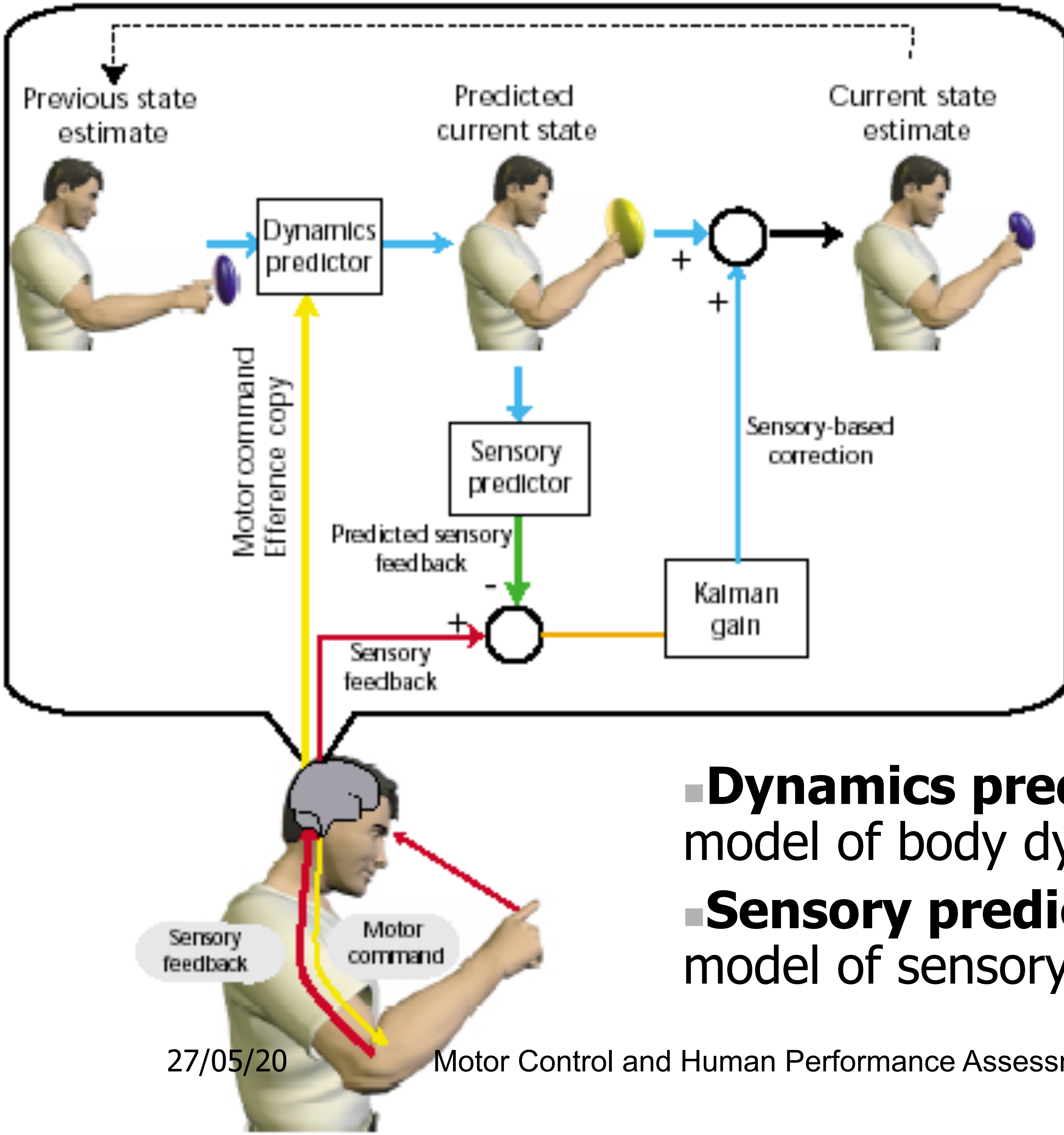
$$\hat{x}_{k+1} = A\hat{x}_k^- + Bu_k + K_{k+1} \cdot \left[y_{k+1} - C\hat{x}_{k+1}^- \right]$$

Forward model (of the body)

Correction

•State observer: 'predicts' future state, combining predictions and sensory information

•Contains a type of 'internal model' (forward model) of the body which is different from those seen until now (inverse models)

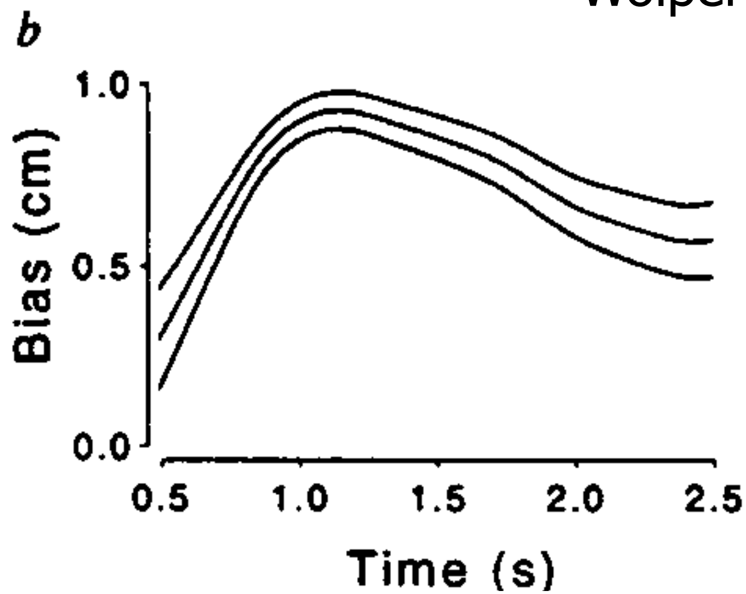


Kalman filter: building blocks

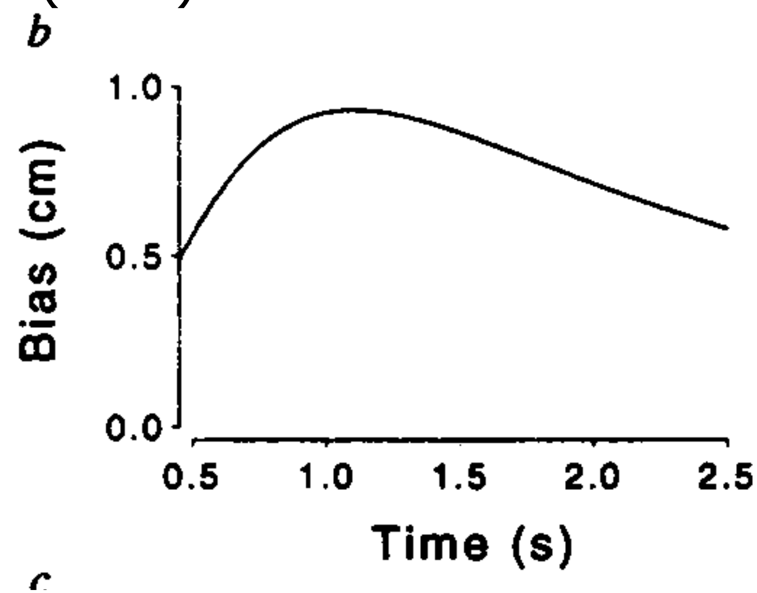
- **Dynamics predictor**: forward model of body dynamics
- **Sensory predictor**: forward model of sensory system

Kalman filter as model of sensorimotor integration

Wolpert et al (1995)



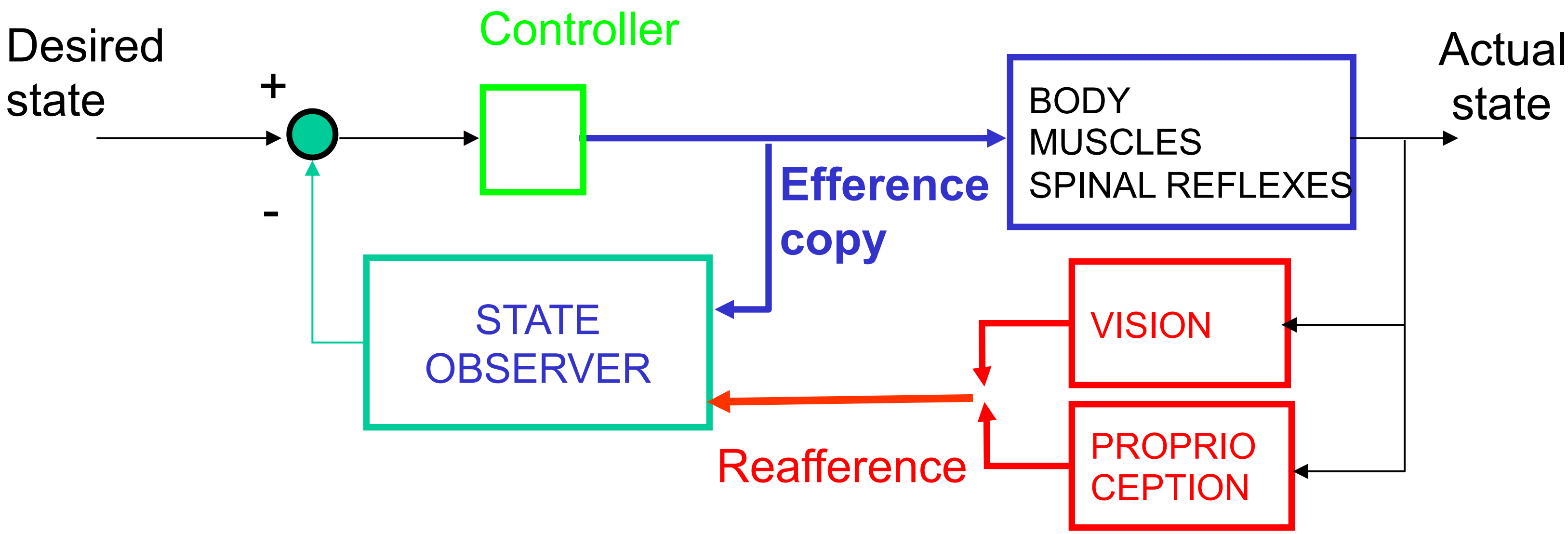
Experiment



Model (Kalman)

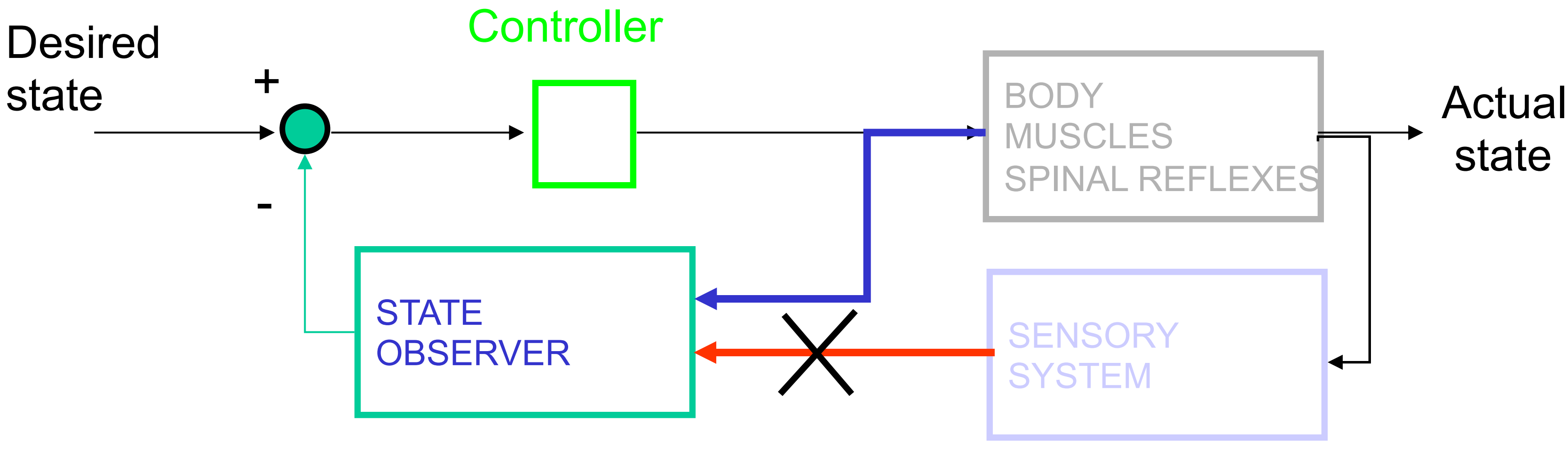
- Early in the movement, contribution of efferent copy prevails: error tends to accumulate
- When afferences come into play, estimate improves and error decreases

Sensorimotor control: model

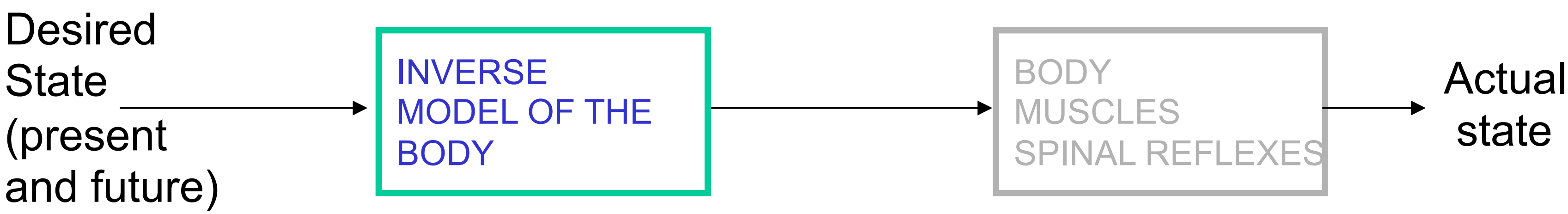


- Feedforward control
  - Experiments suggest rather accurate
- Feedback control
  - State observer needed to overcome sensory delays
  - Multi-sensory integration
  - Sensorimotor integration
- **Alternative models or they can be reconciled?**

Feedback vs feedforward control



- In absence of sensory information (dead reckoning)…
- If the controller has infinite gain:



Feedback controller based on state observer (Kalman) also includes feedforward component

Control modalities: summary

- **Impedance ('postural')**
  - Muscle visco-elasticity
  - Co-contraction
  - Modulation of spinal reflexes
- **Feedforward ('anticipatory')**
  - Requires an 'inverse' internal model of the body
  - Crucial in early phase of movement
  - Problems with inaccuracies and external disturbances
- **Feedback ('corrective')**
  - Uses a 'forward' internal model of body mechanics and sensory system to predict consequences of actions and to make sense of delayed sensory information
  - Also involved in fast movements